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Drag Coefficient and Relative Velocity in Bubbly, Droplet or Particulate Flows

Drag coefficient and relative motion correlations for dispersed two-phase flows of bubbles, drops, and particles were developed from simple similarity criteria and a mixture viscosity model. The results are compared with a number of experimental data, and satisfactory agreements are obtained at wide ranges of the particle concentration and Reynolds number. Characteristic differences between fluid particle systems and solid particle systems at higher Reynolds numbers or at higher concentration regimes were successfully predicted by the model. Results showed that the drag law in various dispersed two-phase flows could be put on a general and unified base by the present method.

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SCOPE

The ability to predict the interfacial drag or the relative velocity between phases is of considerable importance for analyzing a dispersed two-phase system under steady state or transient conditions. For example, the designs and performances of fluidization, sedimentation, and extraction systems as well as various heat transfer systems can be significantly affected by a reliable interfacial drag correlation.

A number of correlations for the relative velocity could be found in the literature. However, most of them are developed from a limited number of experimental data, and, therefore, are applicable only to a certain type of dispersed flows. In the past, different empirical or semi-empirical correlations had been obtained for solid and fluid particle systems at different Reynolds numbers and particle sizes.

The purpose of this article is to develop constitutive relations for the drag force and the relative velocity for bubbly, droplet, and particulate flows by a unified method. Simple drag similarity criteria and mixture viscosity model are introduced in the analysis. Present drag and relative velocity correlations cover all concentration ranges and wide Reynolds number ranges, from the Stokes regime up to the Newton's regime, or the churn-turbulent flow regime. This unified and consistent model gives an improved understanding of the mechanisms of interfacial momentum transfer in dispersed two-phase flows. It can be useful for predicting void fractions, interfacial area, particle residence time, and occurrences of flooding or concentration shock waves.

CONCLUSIONS AND SIGNIFICANCE

It has been shown that the present drag similarity criteria based on the mixture viscosity concept can be successfully applied to develop a relative velocity correlation for bubbly, droplet, and particulate flows. Comparing theoretical predictions to over 1,000 experimental data indicated that satisfactory agreements could be obtained at wide ranges of particle concentration and Reynolds number. For spherical solid particle systems, the data from the Stokes regime up to the Newton's regime within the concentration range of 0 to 0.55 are examined, whereas for fluid particle systems, the distorted particle and churn-turbulent regimes are extensively studied, because of their practical importance. The success of the present correlation at up to the highest concentration range for spherical solid particle systems was accomplished by introducing the maximum packing in the mixture viscosity relation. This was a definite improvement over the existing correlations.

It is also noted that the present model is sufficient up to the foam or dense packing regime, with the concentration ranging from 0.5 to 0.95 for both bubbly and droplet flows. These comparisons indicated that the postulated drag similarity law based on the mixture viscosity concept was appropriate. Therefore, the drag law governing the motions of bubbles, drops and particles in various dispersed two-phase flows can be explained by a unified and consistent model developed here.

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RELATIVE MOTION

Due to the existence of the relative motion of one phase with respect to the other, many two-phase flow problems should be analyzed in terms of two velocity fields. A general transient two-phase flow problem can be formulated by using a two-fluid model or a mixture model (Ishii 1975). In the two-fluid model, each phase is considered separately, hence the model is formulated in terms of two sets of conservation equations governing the balance of mass, momentum, and energy of each phase. In this model, the interfacial momentum interaction terms between two phases (Carrier 1958, Rannie 1962, Delhaye 1968, Vernier and Delhaye 1968, Bouré and Réocruex 1972, Ishii, 1975, Réocreux 1974) should be specified. For a dispersed two-phase flow, there are at least two transient forces acting at the interface in addition to the standard drag force (Basset 1888, Tchen 1947, Zuber 1964), namely, the virtual mass force arising from the inertia effect and the Basset force due to the development of a boundary layer around a particle. A simpler mixture model such as the drift-flux model (Zuber 1967) can be obtained by replacing the two momentum equations by a mixture momentum equation and a relative velocity correlation. Hence, for the two-fluid model formulation, an interfacial drag correlation should be specified, whereas for the mixture model formulation, a relative velocity correlation is required to close the set of equations.

In the chemical engineering field, numerous experimental studies on relative motion have been conducted for fluidization, sedimentation and extraction systems. Also in relation to advanced heat transfer systems, boiling flows as well as gas-liquid flows have been investigated extensively. The reader is referred to the works by Zenz and Othmer (1960), Govier and Aziz (1972), Wallis (1969), Collier (1972), Kutateladze (1972), and Zuber, et al. (1967) for discussions of various experimental data and proposed correlations for the relative motion between phases and void fractions.

Since most of the existing relative velocity correlations have been developed directly from limited sets of experimental data, they are applicable only to a certain type of dispersed two-phase flows. For example, the correlation of Richardson and Zaki (1954), based on the batch fluidization and sedimentation experiments, is applicable to solid particulate flows. Other correlations (Steinour 1944, Hawksly 1951, Hanratty and Bandukwala 1957, Happel 1958, Loeffler and Ruth 1959, Thomas 1963, Zuber 1964, Wen and Yu 1966) are developed on empirical or semi-analytical bases for solid particle systems, and most of them are limited in terms of an applicable Reynolds number range.

For fluid particle systems such as bubbly flows, droplets in gas flows, and droplets in liquid flows, Gaylor et al. (1959), Zuber and Hench (1962), and Zuber and Findlay (1965) presented experimental results and correlations for relative motion. These indicate considerable differences among solid-particulate, bubbly, and droplet flows.

Existing correlations are purely empirical formulas, or they are based on analytical modeling with very limited applicable ranges, in terms of the nature of the particles and the particle Reynolds number. The present analysis considers the drag similarity law in unified form, by taking into account the effect of both the motion and the presence of other particles. Characteristics of the solid or fluid particles are reflected in the single particle rise velocity and mixture viscosity. By assuming a simple similarity criterion between a single particle system and a multi-particle system based on a proper Reynolds number, the drag coefficient and relative velocity relation have been obtained analytically for various dispersed two-phase flows.

MULTIPARTICLE SYSTEM

In analyzing the interfacial force and relative motion between phases, first, the momentum equation for each phase is considered. Under the assumption that both the averaged pressure and stress in the bulk fluid and at the interface are approximately same, the k-phase momentum equation (Ishii 1975) is given by

$$\alpha_k\rho_k\left(\frac{\partial \vec{v}_k}{\partial t}+\vec{v}_k\cdot\nabla\vec{v}_k\right)=-\alpha_k\nabla p_k+\alpha_k\nabla\cdot(\bar{\bar{\tau}}_k+\bar{\bar{\tau}}_k^T)+\alpha_k\rho_k\vec{g}+\vec{M}_{ik}+(\vec{v}_{ki}-\vec{v}_k)\Gamma_k \quad (1)$$

where $\vec{M}_{ik}$ is the generalized interfacial drag force. And the conservation of the mixture momentum requires

$$\sum_k \vec{M}_{ik}=0 \quad (2)$$

which is the modified form of the averaged momentum jump condition.

By neglecting the lift force due to rotations of particles and the diffusion force due to the concentration gradient, the generalized drag force for the dispersed phase may be modeled by a simple form (Zuber 1964) as

$$\vec{M}_{id}=\alpha_d\vec{F}_D/B_d-\frac{1}{2}\alpha_d\frac{(1+2\alpha_d)}{(1-\alpha_d)}\rho_c\frac{d}{dt}(\vec{v}_d-\vec{v}_c)+\frac{9}{2}\frac{\alpha_d}{r_d}\sqrt{\frac{\rho_c\mu_m}{\pi}}\int^t\frac{d}{d\xi}(\vec{v}_c-\vec{v}_d)\frac{d\xi}{\sqrt{t-\xi}} \quad (3)$$

where $\vec{F}_D$, B_d, and μ_m are the standard drag force, volume of a typical particle and mixture viscosity, respectively. The derivative d/dt is the convective derivative relative to $\vec{v}_d$.

The significance of the various terms in the above equation is as follows: The term on the left-hand side is the combined interfacial drag forces acting on the dispersed phase. The first term on the right-hand side is the skin and form drag under the steady state condition. The second term is the force required to accelerate the apparent mass of surrounding phase when the relative velocity changes. The third term is the Basset force and is the effect of acceleration on the viscous drag and the boundary layer development.

Now consider the derivation of the standard drag coefficient and relative velocity correlation for multi-particle systems. In the absence of the wall, and under a steady state condition without phase change ($\Gamma_k=0$), a multiparticle system in an infinite medium essentially reduces to a gravity dominated one-dimensional flow without transient effects. Then the axial component of the momentum equation of Equation (1) for k phase can be written as

$$0=-\alpha_k\frac{dp_m}{dz}-\alpha_k\rho_k g+M_{ik} \quad (4)$$

Here we have also assumed that the surface tension effect on the pressures can be neglected and therefore $p_c=p_d=p_m$. By adding the phase momentum equations and using Equation (2) we obtain

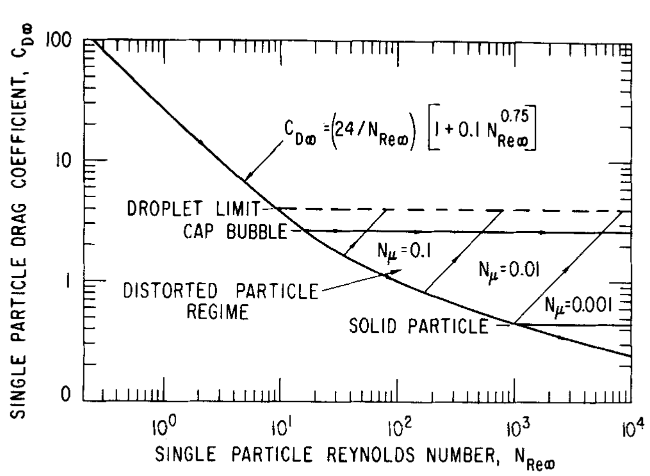

Figure 1. Single particle drag coefficient.

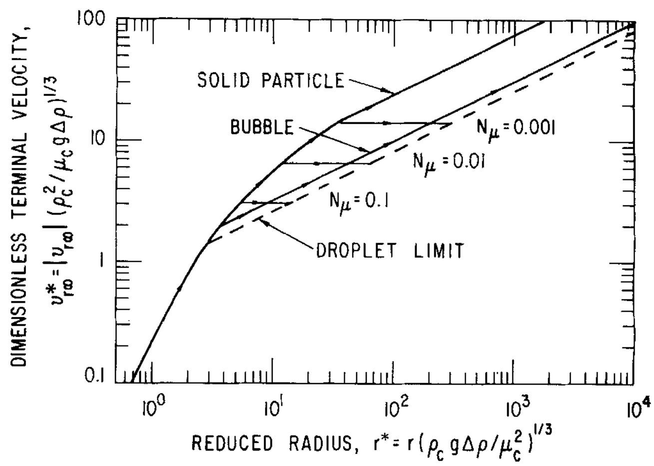

Figure 2. Terminal velocity for single particle system.

$$\frac{dp_m}{dz} = -\rho_m g \tag{5}$$

The drag force acting on the particle under steady-state condition can be given in terms of the drag coefficient C_D based on the relative velocity as

$$F_D = -C_D \tfrac{1}{2}\rho_c v_r |v_r| A_d \tag{6}$$

where A_d is the projected area of a typical particle and v_r is the relative velocity given by $v_r = v_d - v_c$. Then F_D is related to the interfacial force by

$$F_D = M_{id} B_d/\alpha_d \tag{7}$$

where B_d is the volume of a particle. Then from Equations (4) to (7) we obtain

$$v_r|v_r| = \frac{8}{3}\frac{r_d}{C_D\rho_c}(\rho_c - \rho_d)g(1-\alpha_d) \tag{8}$$

where the mean radius of the particle is defined by $r_d = 3B_d/(4A_d)$.

On the other hand, for a system with a single particle in an infinite medium, the force balance as given by Equation (A-2) in a dimensional form is

$$v_{r\infty}|v_{r\infty}| = \frac{8}{3}\frac{r_d}{C_{D\infty}\rho_c}(\rho_c - \rho_d)g \tag{9}$$

Here $v_{r\infty}$ and $C_{D\infty}$ are the terminal velocity and drag coefficient of a single particle in an infinite medium. In general, the drag law for a single particle can be expressed by $C_{D\infty} = C_{D\infty}(N_{Re\infty})$ where the Reynolds Number $N_{Re\infty}$ is given by $N_{R\infty} \equiv 2r_d\rho_c|v_{r\infty}|/\mu_c$. Detailed expressions for $C_{D\infty}$ in various flow regimes are given in the Appendix and shown in Figure 1.

Briefly, for a solid spherical particle system we have the viscous regime where the Reynolds number dependence of $C_{D\infty}$ is pronounced, and the Newton's regime where $C_{D\infty}$ is independent of $N_{Re\infty}$. In case of a clean fluid sphere in the viscous regime, $C_{D\infty}$ can be reduced up to 33% in comparison with the value predicted by the correlation for solid particles. This is explained by the internal circulation within the fluid particles. However, slight amounts of impurities are sufficient to eliminate this drag reduction. For most practical applications, the drag law in a fluid particle system may be approximated by that for a solid particle system, up to a certain particle size. Beyond this point, the distortion of a particle shape and irregular motions become pronounced. In this distorted particle regime, $C_{D\infty}$ does not depend on the viscosity, but it increases linearly with the radius of a particle. Because of the hydrodynamical instability, there is an upper limit on $C_{D\infty}$ and the particle reaches the cap bubble condition or the maximum droplet size. These various regimes for the drag coefficient can be seen in Figure 1. Also, the nondimensional terminal velocity is shown as a function of the dimensionless radius of a particle in Figure 2.

By comparing a single particle system to a multiparticle system, we get from Equation (8) and (9)

$$C_{D\infty}(N_{Re\infty}) = C_D(N_{Re})\left(\frac{v_r}{v_{r\infty}}\right)^2 \frac{1}{1-\alpha_d} \tag{10}$$

Here the Reynolds number is given by

$$N_{Re} = 2r_d\rho_c|v_r|/\mu_m \tag{11}$$

By knowing the mixture viscosity and the dependence of C_D on N_{Re}, Equation (10) can be solved for the velocity ratio $v_r/v_{r\infty}$ to obtain the relative velocity in terms of the single-particle terminal velocity.

The use of Equation (8) and $C_D = C_D(N_{Re})$ in the present analysis is based on the assumption that the resistance experienced by a particle in the two-phase mixture can be evaluated by considering the local resistance to the shearing caused by the relative motion between the representative particle and the surrounding continuous phase. The effect of the presence of other particles arises due to the resistance of the particles to the deformation of flow field.

The use of the mixture viscosity in the similarity group N_{Re} is explained as follows (Burgers 1941 and 1942, Zuber 1964). When a single particle moves through a dispersed two-phase mixture, it imparts a motion to the continuous phase. However, as the fluid moves, its deformation causes the translational and rotational movements of other particles in the neighborhood. Since the particles are more rigid than the fluid against deformations, the particles will impose a system of forces which will react upon the fluid. As a result of additional stresses, the original particle sees an increase in the resistance to its motion which appears as if it arises from an increased viscosity. Consequently, in analyzing the motion of the suspended particles, mixture viscosity should be used. It is expected that the mixture viscosity is a function of the concentration, fluid viscosity and particle viscosity. The viscosity of dispersed phase takes account of the mobility of the interface, and it is the measure of the resistance to the particle material motion along the interface. The effect of the particle collisions may be indirectly reflected in the mixture viscosity through the void fraction. Further, for a fluid particle system, the

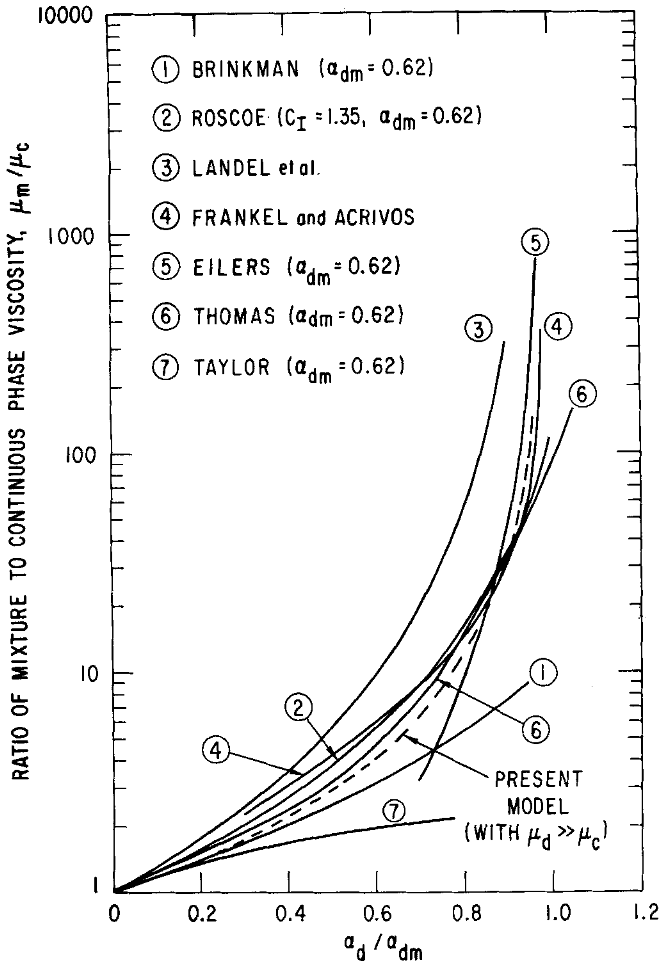

Figure 3. The present mixture viscosity model compared to existing models for solid particle system.

surface tension should affect the particle collisions and coalescences. This should be particularly important in determining the flow regime transitions.

In the present analysis, we extend the linear correlation (Taylor 1932) for the mixture viscosity for fluid particles along the power relation (Roscoe 1952) for solid particles based on the maximum packing α_{dm}. Thus, we have (Ishii 1977)

$$\frac{\mu_m}{\mu_c} = \left(1 - \frac{\alpha_d}{\alpha_{dm}}\right)^{-2.5\alpha_{dm}(\mu_d + 0.4\mu_c)/(\mu_d + \mu_c)} \tag{12}$$

The maximum packing α_{dm} for solid particle systems ranges from 0.5 to 0.74. However, $\alpha_{dm} = 0.62$ suffices for most of the practical cases. For a bubbly flow, theoretical α_{dm} can be much higher because of the deformation of bubbles. In the absence of turbulent motions and particle coalescences, the void fraction in a fluid particle system can be as high as 0.95. By taking α_{dm} to be unity, it is possible to include these foam or dense packing regimes in the analysis. Therefore, for fluid particle systems, we take $\alpha_{dm} = 1$.

Figure 3 compares the present mixture viscosity to the various existing models for solid-particle systems. For the solid particle system μ_d approaches ∞, by taking the limit in Equation (12), the viscosity ratio term becomes unity. By including the effect of viscosity of the dispersed phase in the correlation, the newly developed model has the advantage over the conventional correlations: It is not limited to particulate flows, but can also be applied to droplet and bubbly flows.

VISCOUS REGIME

By assuming that in the viscous regime a complete similarity exists between $C_{D\infty}$ based on $N_{Re\infty}$ and C_D based on N_{Re}, so that the multiparticle drag coefficient C_D has exactly the same functional form in terms of N_{Re} as $C_{D\infty}$ in terms of $N_{Re\infty}$ given by Equation (A-4). Then $C_D = C_{D\infty}(N_{Re})$ or

$$C_D = 24(1 + 0.1\, N_{Re}^{0.75})/N_{Re} \tag{13}$$

The direct substitution of Equation (13) into Equation (10) does not yield a simple relation between v_r and $v_{r\infty}$ due to the implicit form of the equation in terms of v_r. However, an approximate solution for v_r can be obtained by considering the two asymptotic solutions corresponding to $N_{Re} \rightarrow 0$ and $N_{Re} \rightarrow \infty$, then interpolating between these two solutions by a simple function. Thus, an approximate solution to Equation (10) can be given by

$$\frac{v_r}{v_{r\infty}} \doteq (1 - \alpha_d)^{1/2} f(\alpha_d) \frac{1 + 0.1 N_{Re\infty}^{0.75}}{1 + 0.1 N_{Re\infty}^{0.75} [f(\alpha_d)]^{6/7}} \tag{14}$$

where $f(\alpha_d) \equiv (1 - \alpha_d)^{1/2}\mu_c/\mu_m$.

In the drift flux model (Zuber 1967, Ishii et al. 1976), the drift velocity, namely, the relative velocity of the dispersed phase with respect to the volume center of the mixture, is important. This drift velocity V_{dj} can be related to the true relative velocity between phases by

$$V_{dj} = v_d - j = (1 - \alpha_d) v_r \tag{15}$$

Because of its direct physical significance, the drift velocity is used extensively in fluidization, sedimentation and extraction experiments.

By replacing v_r with V_{dj} in Equation (14), we obtain

$$V_{dj} \doteq v_{r\infty}(1 - \alpha_d)^{1.5} f(\alpha_d) \frac{1 + \psi(r_d^*)}{1 + \psi(r_d^*)[f(\alpha_d)]^{6/7}} \tag{16}$$

where $\psi(r_d^*) = 0.55[(1 + 0.08\, r_d^{*3})^{4/7} - 1]^{0.75}$ for the viscous regime, and r_d^* is the nondimensional radius given by $r_d^* = r_d[\rho_c g \Delta\rho/\mu_c^2]^{1/3}$. It is noted here that $N_{Re\infty}$ in Equation (14) has been replaced by a new function ψ using the identity $N_{Re\infty} = 2r_d^* v^*_{r\infty}$ and the relation between $v^*_{r\infty}$ and r_d^* given by Equation (A-6).

The similarity criterion given by $C_D(N_{Re}) = C_{D\infty}(N_{Re})$ with the Reynolds number based on the mixture viscosity is first introduced for solid particle system in the Stokes regime (Hawksley 1951, Zuber 1964). In this case, the velocity ratio reduces to $(1 - \alpha_d)\mu_c/\mu_m$ which is the limiting case of Equation (14), with $N_{Re\infty} << 1$ or $r_d^* << 1$. However, the present theory is not limited to a solid particle system or to the Stokes regime, because the generalized drag law and the mixture viscosity model are used in the analysis.

NEWTON'S REGIME

For a solid-particle system, we assume that the transition from the viscous regime to the Newton's regime occurs at the same radius as in the single-particle system, and that the drag coefficient C_D is a continuous function. This point is not confirmed, but the experimental data of Richardson and Zaki (1954) support this trend. Then for Newton's regime given by $r^* \geqq 34.65$ (see Appendix), we obtain by substituting $r^* = 34.65$ into Equation (16)

$$V_{dj} = v_{r\infty}(1 - \alpha_d)^{1.5} f(\alpha_d) \frac{18.67}{1 + 17.67[f(\alpha_d)]^{6/7}} \tag{17}$$

For a fluid particle system in the viscous regime, i.e., in the undistorted particle regime, the particle behaviors should be quite similar to those of solid particle systems. The main difference is that in a fluid particle system, the physical properties of both phases have significant contributions on the size of particles. Further, the maximum packing, α_d is significantly larger for bubbly or droplet flows than for particulate flows. Equation (16) with a proper mixture viscosity and maximum packing α_{dm} should be used.

DISTORTED FLUID PARTICLE REGIME

In the distorted-fluid-particle regime, the single particle drag coefficient depends only on the particle radius and fluid properties and not on the velocity or the viscosity, i.e., $C_{D\infty} = (4/3)r_d\sqrt{g\Delta\rho/\sigma}$, as discussed by Harmathy (1960). Thus, for a particle of a fixed diameter, $C_{D\infty}$ becomes constant. In considering the drag coefficient for a multiparticle system with the same radius, it is necessary to take into account the restrictions imposed by the existence of other particles on the flow field. Therefore, C_D is expected to be different from $C_{D\infty}$, in this regime. Because of the wake characteristic of the turbulent eddies and particle motions, a particle sees the increased drag due to other particles in essentially similar ways as in the Newton's regime for a solid-particle system, where $C_{D\infty}$ is also constant under a wake turbulent flow condition. Hence, we postulate that regardless of the differences in $C_{D\infty}$ in these regimes, the effect of increased drag in the distorted-fluid-particle regime can be predicted by the similar expression as that in the Newton's regime. In other words, we assume that $C_D/C_{D\infty}$ for the distorted particle regime is the same as that in the Newton's regime.

Under this assumption, Equation (17) can be used for the distorted-particle regime with the appropriate $v_{r\infty}$. Substituting the definition of $f(\alpha_d)$ into Equation (17), the results can be further simplified to

$$V_{dj} \doteq v_{r\infty} \times \begin{cases} (1-\alpha_d)^{1.75}; & \mu_c >> \mu_d \\ (1-\alpha_d)^2\ ; & \mu_c \sim \mu_d \\ (1-\alpha_d)^{2.25}; & \mu_d >> \mu_c \end{cases} \tag{18}$$

where for the distorted-fluid-particle regime $|v_{r\infty}| = \sqrt{2}(g\sigma\Delta\rho/\rho_c^2)^{0.25}$. The above criterion is applicable for $N_\mu \geqq 0.11(1+\psi)/\psi^{8/3}$, where the viscosity number is given by $N_\mu \equiv \mu_c/(\rho_c\sigma\sqrt{\sigma/g\Delta\rho})^{0.5}$. It is noted that a similar dimensionless group has been used by various researchers (Hinze 1955, Wallis 1969, Ishii and Grolmes 1975). Basically, it scales the fluid viscous force against the surface tension force, and it is a simple property group. For a distorted-bubbly flow, the first expression under the condition $\mu_c >> \mu_d$ is applicable, which is very close to the empirical correlation of Zuber and Findley (1965), that is $V_{dj} = v_{r\infty}(1-\alpha_d)^{1.5}$. It is also noted here that Equation (18) under the condition $\mu_d >> \mu_c$ can be used for the Newton's regime with $|v_{r\infty}| = 2.43\sqrt{gr_d\Delta\rho/\rho_c}$, if the solid particle concentration is not close to the maximum packing α_{dm}.

CHURN-TURBULENT REGIME

As the radius of the fluid particle is further increased, the wake and bubble boundary layer can overlap due to the formation of a large wake regions. In other words, a particle can influence both the surrounding fluid and other particles directly, and the entrainment of a particle in a wake of other particles becomes possible. This flow regime is known as the churn-turbulent flow regime and is commonly observed in bubbly flows. In the existence of sufficient turbulent motions in the continuous phase, the transition from the distorted particle regime to the churn-turbulent flow regime occurs at the particle concentration around 0.3. This criterion for the transition can be applied to most of forced convection two-phase flows. However, in case of a batch process, detailed coalescense mechanisms and surface contaminations become important in determining the transition criterion.

In the churn-turbulent flow regime, a typical particle moves with respect to the average volumetric flux j rather than the average velocity of a continuous phase because of the hydrodynamic conditions discussed above. Hence, the reference velocity in the definitions of the drag coefficient and the drag similarity law should be the drift velocity rather than the relative velocity. Hence, the drag force should be given by

$$F_D = -C_D' \tfrac{1}{2}\rho_c V_{dj}|V_{dj}|\pi r_d^2 \tag{19}$$

In a churn-turbulent flow regime, some particles should have reached the distortion limit corresponding to the cap-bubble transition or the droplet disintegration. This limit can be given as an extension of the Weber number criterion (Wallis 1969), by using the drift velocity as a reference velocity in the following form

$$\frac{2\rho_c V_{dj}^2 r_d}{\sigma} = \begin{cases} 8\ \text{(bubble)} \\ 12\ \text{(droplet)} \end{cases} \tag{20}$$

Due to the entrainment of particles in a wake of other larger particles and the coalescence and disintegration caused by the turbulence, the average motion of the dispersed phase is mainly governed by these particles which satisfy the Weber number criterion. The effective drag coefficient is given by $C_D' = 8/3$. If we recast the above drag force expression based on the drift velocity to the conventional one based on the relative velocity we obtain

$$F_D = -\frac{8}{3}(1-\alpha_d)^2 \frac{\rho_c v_r|v_r|\pi r_d^2}{2} \tag{21}$$

which gives the apparent drag coefficient $C_D = (8/3)(1-\alpha_d)^2$ in this regime.

From Equation (21) it is straight forward to obtain the drift velocity by balancing the drag force with the pressure and gravity forces, thus we have

$$V_{dj} = \begin{Bmatrix} \sqrt{2} \\ \text{or } 1.57 \end{Bmatrix} \left(\frac{\sigma g\Delta\rho}{\rho_c^2}\right)^{1/4} \left(\frac{\rho_c-\rho_d}{\Delta\rho}\right)(1-\alpha_d)^{1/4}$$
$$\doteq \sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_c^2}\right)^{1/4}\left(\frac{\rho_c-\rho_d}{\Delta\rho}\right) \tag{22}$$

In the first expression for V_{dj} the proportionally constant $\sqrt{2}$ is applicable for bubbly flows and 1.57 for droplet flows. However, in view of the uncertainty in predicting the drag coefficient, this difference, as well as the effect of the void fraction, may be neglected. The above result is consistent with the study of Zuber and Findley (1965) for bubbly flows.

APPLICATIONS AND COMPARISONS WITH EXPERIMENTAL DATA

Figure 4 compares the present analytical results given by Equations (16) and (17) with the empirical correlation for solid-particle flow systems (Richardson and Zaki

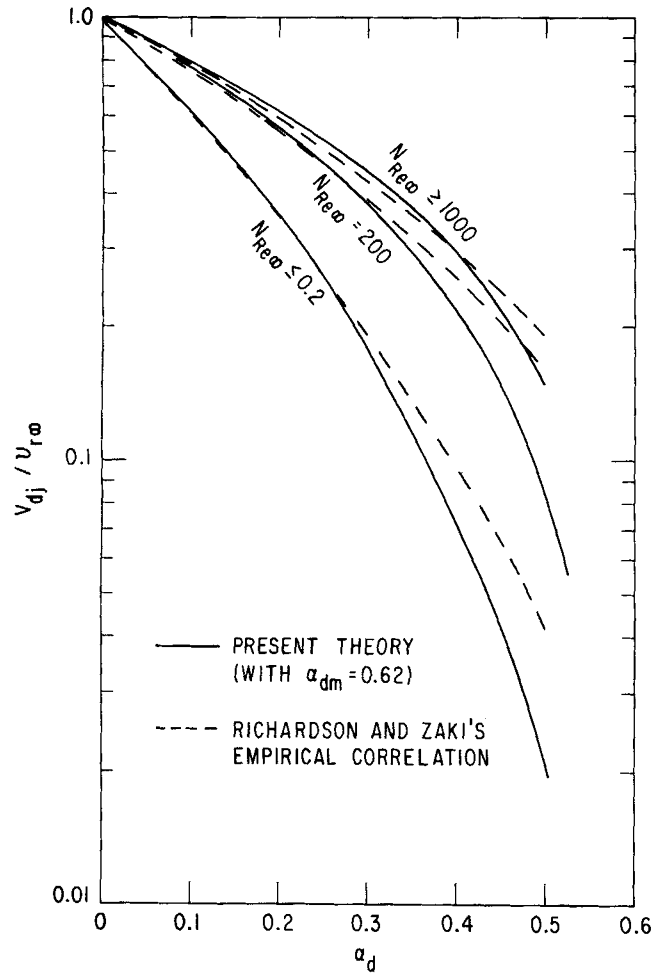

Figure 4. Predicted results compared with Richardson and Zaki's empirical correlations for solid particulate flow system.

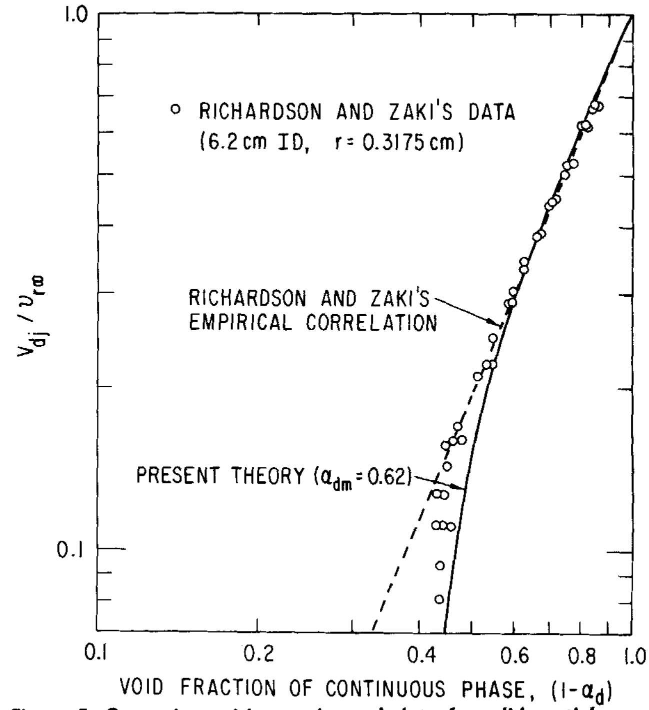

Figure 5. Comparison with experimental data for solid particle system at high Reynolds numbers.

1954). Agreement at relatively low volumetric concentrations is excellent at all Reynolds-number regions. At very high values of α_d, the present theory predicts much lower drift velocities than the Richardson-Zaki correlation. However, the original experimental data of Richardson and Zaki also indicate this trend, which is predicted by

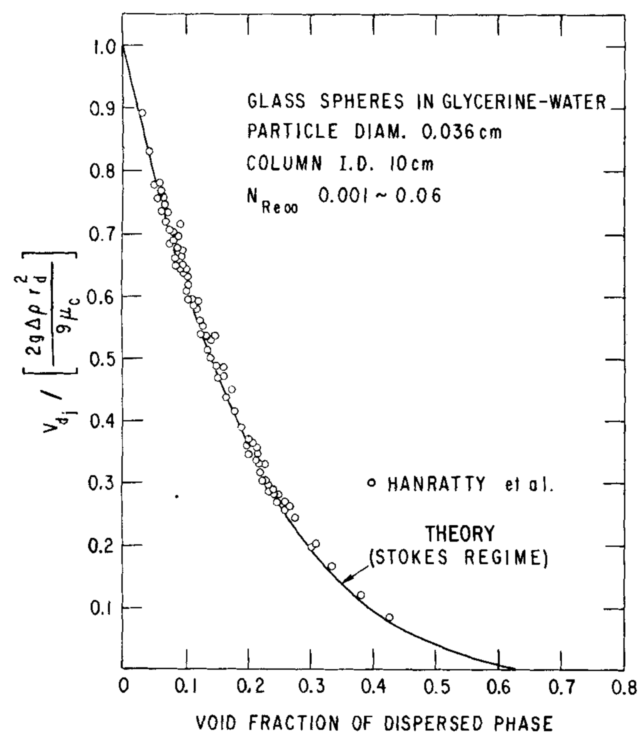

Figure 6. Comparison with experimental data for solid particle-liquid systems at low Reynolds numbers.

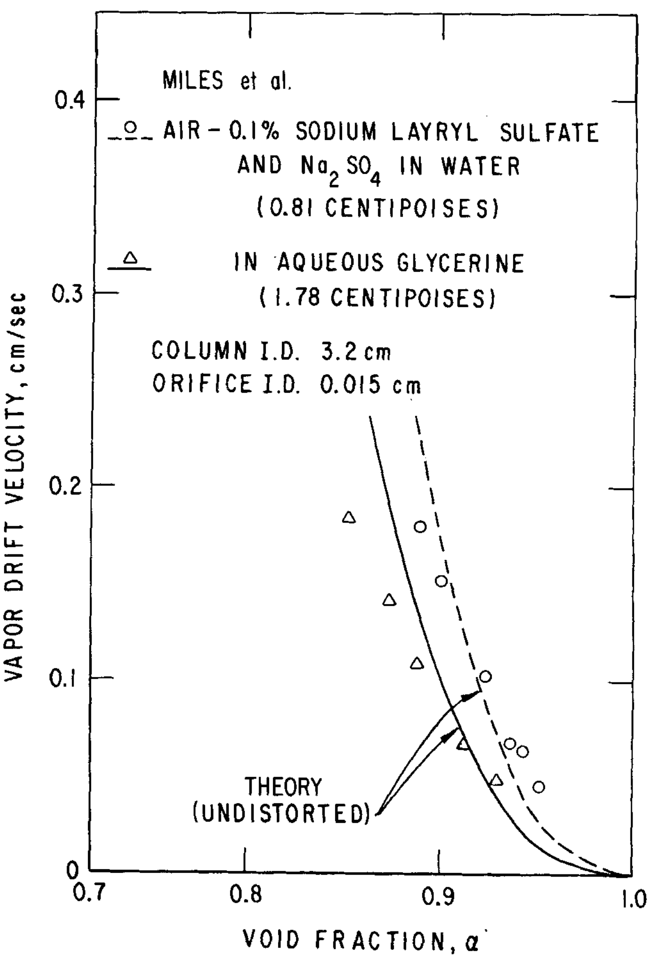

Figure 7. Undistorted bubbly flow regime drift velocity compared to experimental data of Miles et al., at very high void fraction regimes.

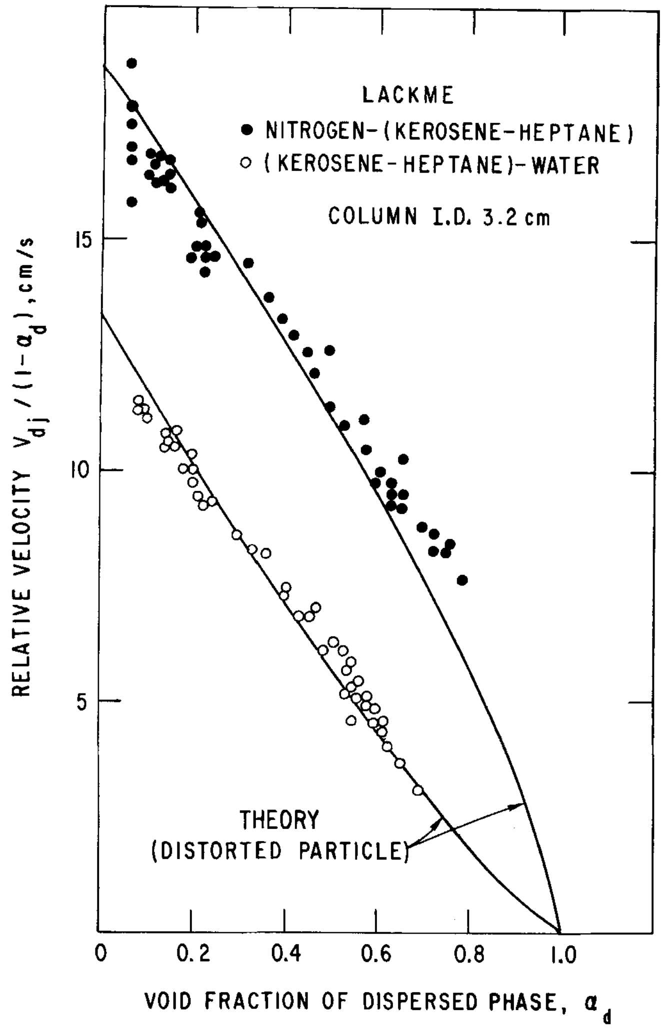

Figure 8. Difference between bubble system and droplet dispersion system, in distorted particle regime.

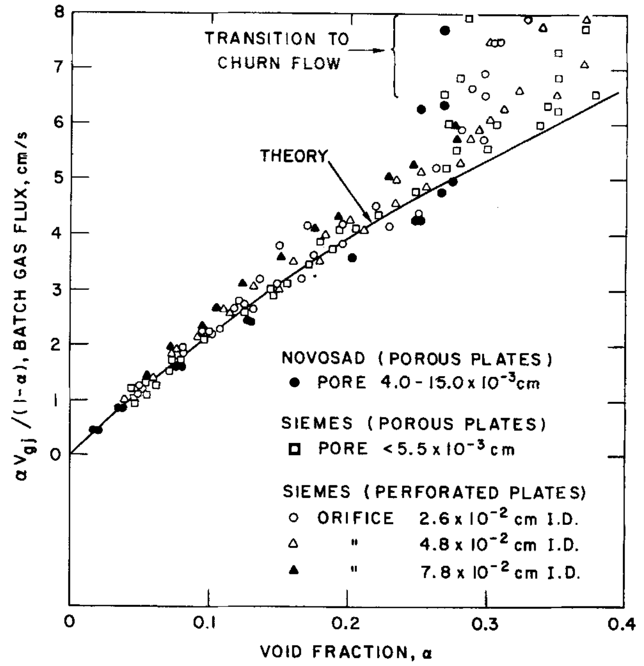

Figure 9. Predicted gas flux under batch operation compared with air-water experimental data.

the present theory as shown in Figure 5. The data shown are taken at relatively high Reynolds numbers, however, the similar behavior occurred at the lower Reynolds number regions. For example, the experimental data of Hanratty and Bandukwala (1957) obtained in the Stokes regime agreed with the present theory at all concentrations (see Figure 6).

At maximum packing, the particles will interlock each other and therefore, cannot be suspended as individual particles in fluid. The drift velocity should become very small as the particle concentration approaches the maximum packing α_{dm}. The present model correctly predicted the relative motion up to this highest concentration over wide ranges of the Reynolds number.

A particular interest in this undistorted-fluid-particle regime is the higher concentration region, namely, the foam or dense packing region. There is a pronounced difference between fluid and solid particle systems that can be observed due to the difference in the maximum packing. In Figure 7, the experimental data of Miles et al. (1945) are compared to our model. Because of the direct particle to particle interactions in the foam region, some discrepancies between the predictions and the data are expected. However, in spite of these effects, the agreement appears surprisingly satisfactory. Results for solid and fluid particle systems can be considered as indirect support for the assumed similarity criterion for the drag coefficient based on the multiparticle Reynolds number.

Results for the distorted particle regime given by Equation (18) have been compared to various experimental data for bubbly flows (Lackme 1973, Bridge et al. 1964, Novosad 1954, Siemes 1954) and for droplet-liquid flows (Kutateladze and Moskvicheva 1960, Lackme 1973, Weaver et al. 1959, Letan and Kehat 1967). Figure 8 shows the relative velocity both in the bubbly

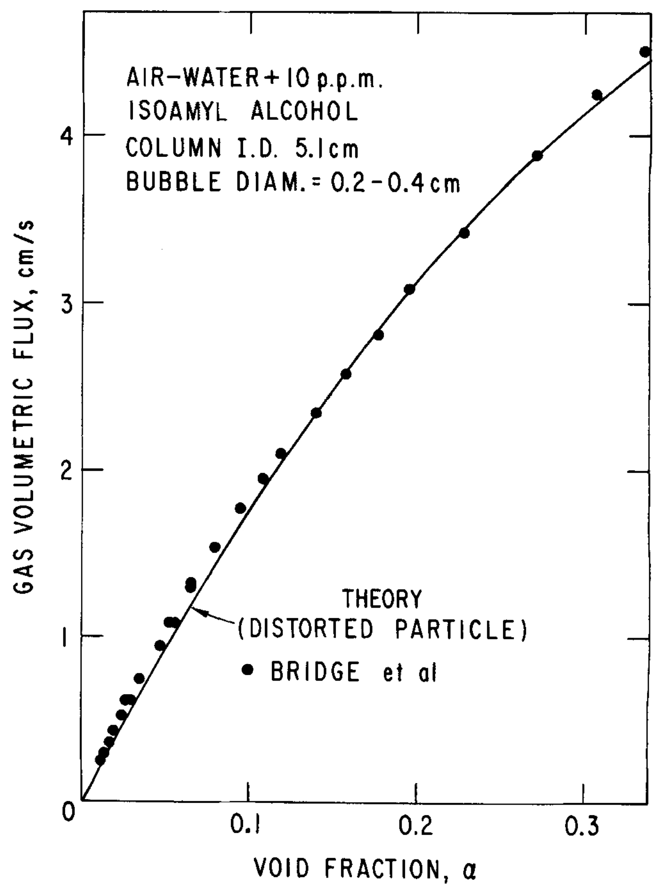

Figure 10. Gas volumetric flux compared with distorted bubble regime data.

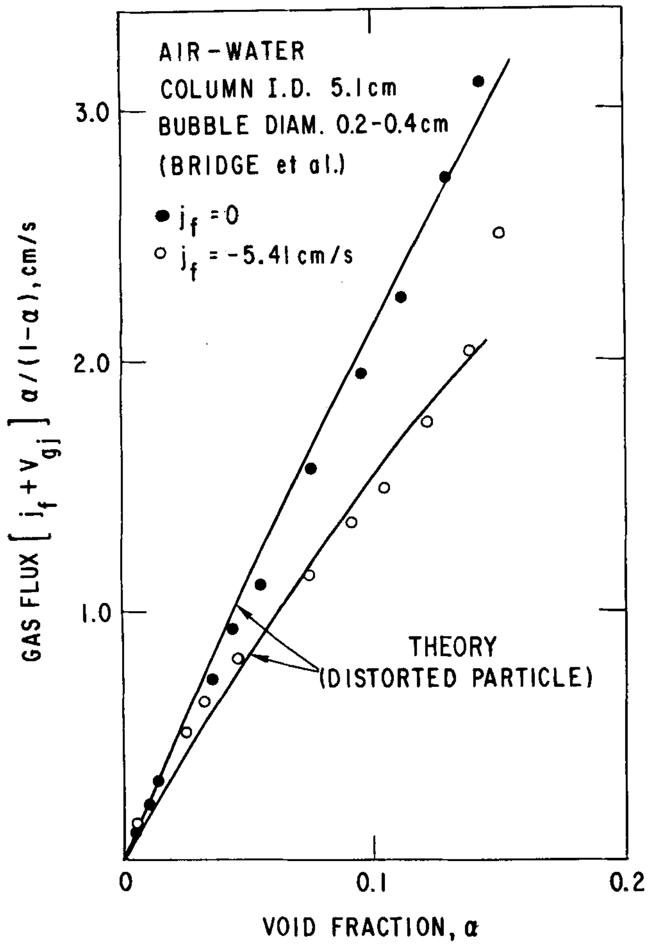

Figure 11. Gas volumetric flux compared with distorted bubble regime data.

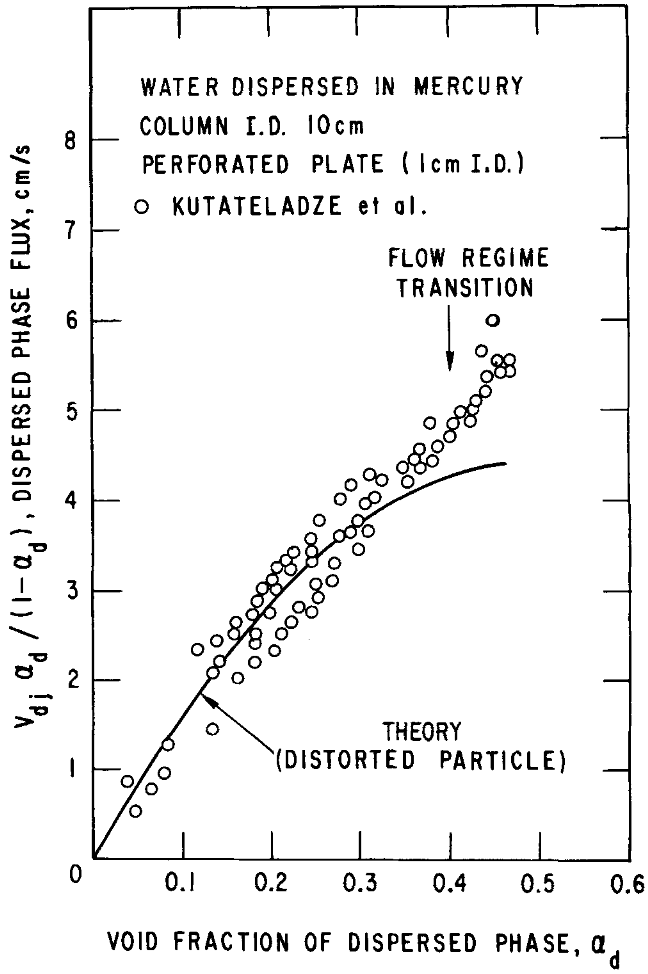

Figure 12. Comparison of the predicted liquid volumetric flux, based on the distorted particle regime drift velocity, to data.

and the droplet-liquid flow regimes. These data of Lackme clearly indicate the difference in the concentration dependence of the relative velocity between a bubbly flow and a droplet flow. These characteristics have been correctly predicted by Equation (18). Further comparisons between the theoretical predictions and experimental data, both in the batch and countercurrent bubbly flows, are given in Figures 9, 10, and 11. In Figure 9, the

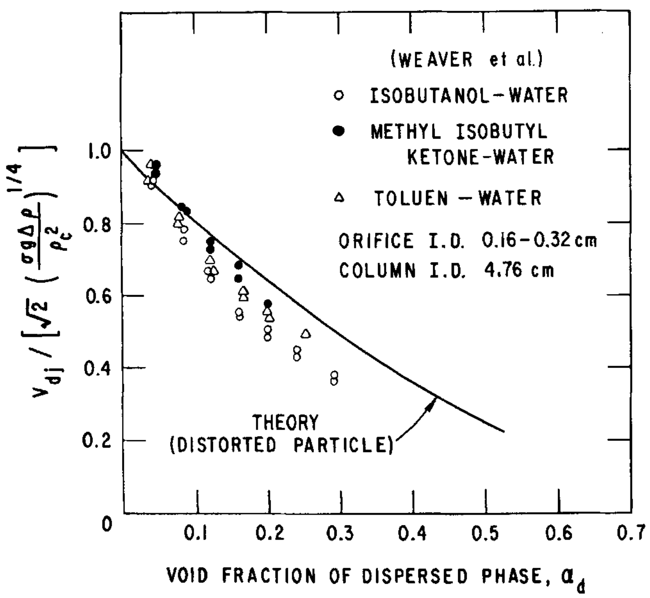

Figure 13. Comparison of the predicted distorted particle regime drift velocity to data.

departure of the experimental data from the prediction around void fraction 0.3 can be observed. This indicates the flow regime transition due to increased turbulence and bubble coalescences. The new flow regime is either the churn turbulent or slug flow regime and it contains considerably larger bubbles.

Comparisons between the present theory and results of various liquid-liquid dispersion experiments are shown in Figure 12, 13, and 14. In case of bubbling water through mercury (Kutateladze and Moskvicheva 1960) the flow regime transition occurs at $\alpha_d \doteq 0.4$. However, as can be seen from Figures 8 and 14, the distorted particle regime can exist in much higher void fraction ranges in the absence of significant amount of coalescences among fluid particles. Except the data of Weaver et al. (1959) the agreements of the theoretical predictions with the data for various fluid combinations are satisfactory. The discrepancies between the model and the data of Weaver may be due to contaminations of the interfaces.

An interesting case of the bubble dispersion in the flowing liquid is shown in Figure 15. Serizawa (1974) measured the local void fraction and velocities. The data shown are the plot of the local relative velocity versus the local void fraction. According to our model, the undistorted particle (viscous regime) correlation given by Equation (16) applies to this case. However, as a reference the distorted particle correlation, Equation (18), is also shown in Figure 15. The relative velocity is a small quantity, and it has been obtained from the difference between two larger quantities, i.e., the phase velocities. This is reflected in the large scattering of

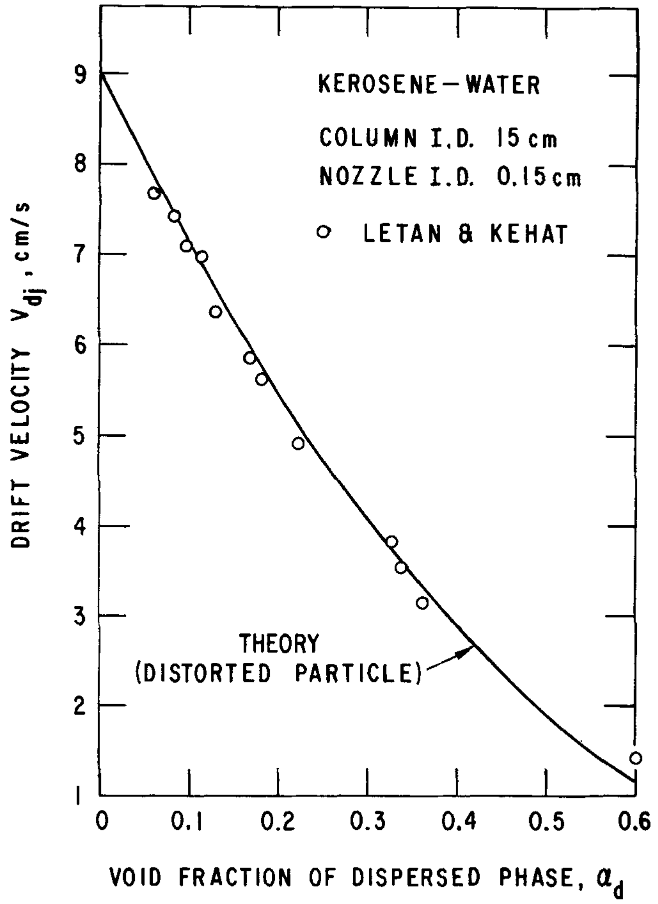

Figure 14. Comparison of the predicted distorted particle regime drift velocity to data.

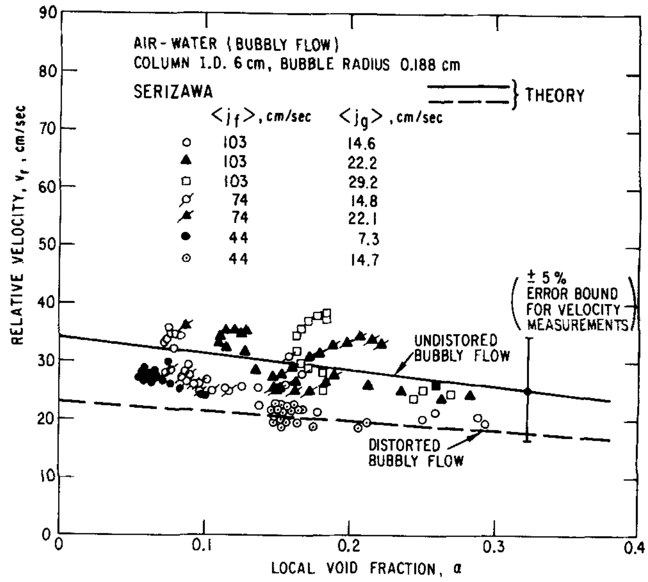

Figure 15. Comparison of the predicted relative velocity to the local measurements of Serizawa (1974).

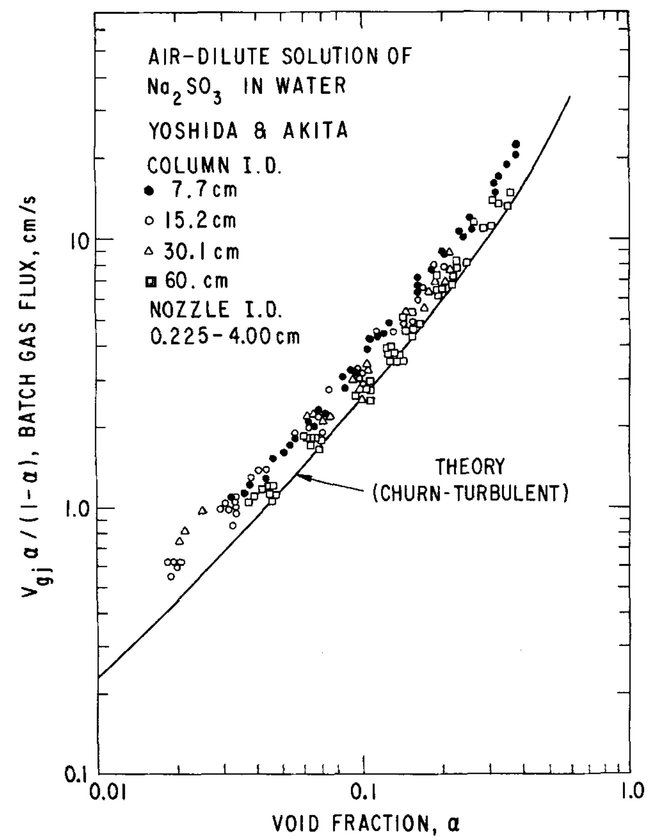

Figure 16. Comparison of the predicted gas volumetric flux, based on the churn flow regime drift velocity, to data.

the data around the theory. Although our model agrees with the data within ±5% of the measurement errors, some observations on the two dimensional effect can be made. Data taken near the centerline of the tube tend to follow the prediction based on the undistorted particle correlation. However, data from the near wall region approach the distorted particle regime correlation. This can be explained by the distortion of bubbles near the wall due to the high liquid shear, which has not been accounted in the flow regime transition criterion. The existence of two flow regimes within a single cross section explains the two peaks of the local void fraction observed in the experiment.

A comparison between the prediction for a churn-turbulent flow based on Equation (22) and the experimental data of Yoshida and Akita (1965) is shown in Figure 16. Data were taken for an air-aqueous sodium sulfite solution system with various column diameters ranging from 7.7 cm to 60 cm. As can be seen, the theory underpredicts the gas flux for smaller column diameter experiments. However, for larger column diameter cases, agreement between the prediction and the data becomes increasingly satisfactory. This tendency can be easily explained by the two-dimensional effect due to the void and velocity profiles.

The dispersed phase is locally transported with the local drift velocity with respect to local volumetric flux. Therefore, if more particles are concentrated in higher flux regions, this will give a higher dispersed-phased flux than the case with uniform profiles. Then, the mean gas volumetric flux should be somewhat higher than the prediction. For more detailed discussions on this point, the readers are referred to Zuber and Findley (1965) and Werther (1974).

SUMMARY AND DISCUSSIONS

The present drag similarity criterion $C_D(N_{Re})$ and the mixture viscosity concept $\mu_m(\alpha_d)$ have been used to develop a relative motion between phases for bubbly, droplet and particulate flows. The results are summarized in Table 1. Comparisons of the theoretical predictions to over 1,000 experimental data points show that satisfactory agreements are obtained at all ranges of particle concentration and Reynolds number.

Table 1. Summary of Drift Velocity V_{dj} in Infinite Media

Regime: \ μ_c/μ_m	Bubble in liquid	Droplet in liquid	Droplet in gas	Solid particle
μ_c/μ_m	$(1-\alpha_d)$	$\sim(1-\alpha_d)^2$	$\sim(1-\alpha_d)^{2.6}$	$\sim(1-\alpha_d)^{2.6}$
Stokes regime:	$\frac{2}{9}\frac{g\Delta\rho r_d^2}{\mu_f}(1-\alpha_d)^3$		$\frac{2}{9}\frac{g(\rho_c-\rho_d)}{\mu_c} r_d^2 (1-\alpha_d)^2 \frac{\mu_c}{\mu_m}$	
Undistorted particle regime: $N_\mu \lesssim 0.11 \frac{1+\psi}{\psi^{8/3}}$		$\frac{10.8\,\mu_c}{\rho_c r_d}\frac{\mu_c}{\mu_m}(1-\alpha_d)^2 \frac{\psi^{4/3}(1+\psi)}{1+\psi\left[\frac{\mu_c}{\mu_m}(1-\alpha_d)^{0.5}\right]^{6/7}} \frac{\rho_c-\rho_d}{\Delta\rho}$ where $\psi = 0.55[(1+0.08\,r_d^{*3})^{4/7}-1]^{0.75}$		for Newton's regime ($r_d^* \geqq 34.67$) $\psi = 17.67$
Distorted particle regime:	$n = 1.75$	$\sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_c^2}\right)^{1/4}(1-\alpha_d)^n \frac{\rho_c-\rho_d}{\Delta\rho}$; $n \sim 2.0$	$n = 2.25$	
Churn turbulent regime:	$\sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_f^2}\right)^{1/4}$	$\sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_c^2}\right)^{1/4}\frac{\rho_c-\rho_d}{\Delta\rho}$		

The range of the volumetric concentration considered in the comparison extends from $\alpha_d = 0$ up to $\alpha_d = 0.55$ for solid particle systems. For fluid particle systems, even higher concentration data are included. In particular, the dense packing regime of α_d up to 0.95 is examined for the case of the droplet suspension in liquid.

In terms of the particle Reynolds number, data in the comparison spanned from the Stokes regime all the way up to the Newton's regime for solid particle systems. For fluid particle systems, data for the distorted particle regime and the churn-turbulent regime are used extensively to study the difference between the solid and fluid particle systems.

These results showed that the drag law in various dispersed two-phase flows could be put on a general and consistent base through the use of the mixture viscosity. The successful prediction of the relative velocity for solid particles as well as for fluid particles in a wide range of Reynolds number by the unified drag law can be considered as an indirect verification of the drag similarity criterion. Consequently, the drag coefficient for multiparticle systems in the form defined by Equations (3) and (6) and postulated in the previous section may be accepted as a reliable correlation. The summary of the present drag coefficient in various flow regimes is given in Table 2.

The present relative velocity correlation and the formula for the drag coefficient for multiparticle systems have been developed from the steady-state and adiabatic formulation. It is postulated that the transient effect on the momentum exchange term could be taken into account by an essentially linear constitutive relations in the form of Equation (3). Therefore, it is indirectly assumed that the standard drag coefficient developed in the analysis

Table 2. Summary of Drag Coefficient in Multiparticle System

	Solid particle system	Fluid particle system
$\frac{\mu_m}{\mu_c}$:	$\frac{\mu_m}{\mu_c} = \left(1-\frac{\alpha_d}{\alpha_{dm}}\right)^{-2.5\alpha_{dm}(\mu_d+0.4\mu_c)/(\mu_d+\mu_c)}$	
Stokes regime:	$C_D = \frac{24}{N_{Re}}$ where $N_{Re} = 2r_d\rho_c v_r/\mu_m$	
Undistorted particle regime (viscous regime):	$C_D = \frac{24}{N_{Re}}(1+0.1\,N_{Re}^{0.75})$	
Newton's regime: $r_d\left(\frac{\rho_c g\Delta\rho}{\mu_c}\right)^{1/3} \geqq 34.65$	$C_D = 0.45\left\{\frac{1+17.67\,[f(\alpha_d)]^{6/7}}{18.67\,f(\alpha_d)}\right\}^2$ where $f(\alpha_d) = \sqrt{1-\alpha_d}\left(\frac{\mu_c}{\mu_m}\right)$	
Distorted particle regime: $N_\mu \geqq 0.11 + \frac{1+\psi}{\psi^{8/3}}$		$C_D = \frac{4}{3} r_d \sqrt{\frac{g\Delta\rho}{\sigma}}\left\{\frac{1+17.67\,[f(\alpha_d)]^{6/7}}{18.67\,f(\alpha_d)}\right\}^2$ or $C_D = \frac{4}{3} r_d \sqrt{\frac{g\Delta\rho}{\sigma}} \times \begin{cases}(1-\alpha_d)^{-0.5}; & \mu_c >> \mu_d \\ (1-\alpha_d)^{-1}; & \mu_c \sim \mu_d \\ (1-\alpha_d)^{-1.5}; & \mu_c >> \mu_d\end{cases}$
Churn turbulent flow regime:		$C_D = \frac{8}{3}(1-\alpha_d)^2$

could also be used under transient conditions. The additional interfacial forces due to the inertia effect and development of boundary layer in a transient flow are considered separately.

The phase change at the particle surface contributes to the interfacial momentum transfer in two different ways. There is a direct effect of momentum carried by the mass undergoing phase change as represented by the last term of Equation (1). Changing particle size or shape due to phase change, and the modification of the boundary layer around the particles by additional mass flux normal to the surface, may affect the standard drag coefficient. However, the effects of heat transfer and phase changes are considered as secondary here. These effects appear only indirectly through the local variables such as the void fraction, particles sizes and component velocities. To assess the significance of the phase change effect on the drag coefficient, further experimental and analytical studies are required.

ACKNOWLEDGMENT

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NOTATION

A_d = projected area of a particle
B_d = volume of a particle
C_D = drag coefficient for multiparticle system
$C_{D\infty}$ = drag coefficient for a single particle system
$f(\alpha_d)$ = given function of void fraction
F_D = drag force
F_p = pressure force
F_g = gravity force
g = gravity
j = mixture volumetric flux
j_k = volumetric flux of k phase
M_{ik} = interfacial force of k phase
N_μ = viscosity number based on continuous phase viscosity
$N_{Re\infty}$ = single particle Reynolds number
N_{Re} = particle Reynolds number
p = pressure
r_d = particle radius
r_d^* = non-dimensional radius
t = time
V_{dj} = local drift velocity
v_m = mixture velocity
v_k = velocity of k phase
v_r = relative velocity, $v_d - v_c$
$v_{r\infty}$ = terminal velocity for a single particle system
v^* = non-dimensional velocity
z = axial coordinate

Greek Letters

α_k = void fraction of k phase
α_{dm} = maximum packing void fraction
Γ_k = mass source for k phase
$\Delta\rho$ = absolute value of density difference
μ_k = viscosity of k phase
μ_m = mixture viscosity in dispersed flow
ρ_k = density of k phase
ρ_m = density of mixture
σ = interfacial tension
$\bar{\bar{\tau}}_k$ = averaged viscous stress for k phase
$\bar{\bar{\tau}}_k^T$ = turbulent stress for k phase
ψ = given function of r_d^*

Subscripts

c = continuous phase
d = dispersed phase
f = liquid phase
g = vapor or gas phase
k = k phase ($k = c, d$ or $k = g, f$)
ki = k phase at interface
m = mixture

APPENDIX: RELATIVE MOTION IN SINGLE PARTICLE SYSTEM

A motion of the single solid particles, drops, or bubbles in an infinite medium has been studied extensively in the past, see for example, Peebles and Garber (1953), Harmathy (1960) and Wallis (1974). In what follows we shall summarize these results in simple forms useful for the development of the drift constitutive equation in multiparticle systems.

By denoting the relative velocity of a single particle in an infinite medium by $v_{r\infty} = v_d - v_{c\infty}$, the drag coefficient is defined by $C_{D\infty} \equiv -2F_D/\rho_c v_{r\infty}|v_{r\infty}|\pi r_d^2$ where F_D is the drag force and r is the radius of a particle. On the other hand the pressure and body forces acting on the particle is given by

$$F_P + F_g = \frac{4}{3}\pi r_d^3 (\rho_c - \rho_d) g \tag{A-1}$$

which should be balanced by the drag force. Hence we have $F_p + F_g + F_D = 0$. By introducing the nondimensional parameters for the velocity fields and radius given by $v^* \equiv |v|(\rho_c^2/\mu_c g\Delta\rho)^{1/3}$ and $r_d^* \equiv r_d(\rho_c g\Delta\rho/\mu_c^2)^{1/3}$, the force balance can be solved for the radius as

$$r_d^* = \frac{3}{8} C_{D\infty} v_{r\infty}^{*2} \tag{A-2}$$

The standard particle Reynolds number and the viscosity number are defined by

$$\left.\begin{aligned} N_{Re\infty} &\equiv \frac{2 r_d \rho_c |v_{r\infty}|}{\mu_c} = 2 r_d^* v^*_{r\infty} \\ N_\mu &\equiv \mu_c \Big/ \left(\rho_c \sigma \sqrt{\frac{\sigma}{g\Delta\rho}}\right)^{1/2} \end{aligned}\right\} \tag{A-3}$$

Extensive studies on the single particle drag show that in general the drag coefficient is a function of the Reynolds number. However, the exact functional form depends on whether the particle is a solid particle, drop, or bubble.

For viscous regime, the function C_D is given by

$$C_D = \frac{24}{N_{Re\infty}} (1 + 0.1\, N_{Re\infty}^{0.75}) \tag{A-4}$$

In the case of solid particles, the drag coefficient becomes essentially constant at approximately $C_{D\infty} = 0.45$ for $N_{Re\infty} \geqq 1000$. This Newton's regime holds up to $N_{Re\infty} = 2 \times 10^5$.

For fluid particles such as drops or bubbles, we have a flow regime characterized by the distortion of particle shapes and irregular motions. In this distorted particle regime, the experimental data show that terminal velocity is independent of the particle size. The drag coefficient $C_{D\infty}$ does not depend on the viscosity, but it should be proportional to the radius of the particle (Harmathy 1960). Physically, the drag force is governed by distortion and swerving motion of the particle, and change of the particle shape is toward an increase in the effective cross section. Therefore, $C_{D\infty}$ should be scaled by the mean radius of the particle rather than the Reynolds number (Harmathy 1960), then we have

$$C_{D\infty} = \frac{4}{3} r_d \sqrt{g\Delta\rho/\sigma} \left(\text{or } C_{D\infty} = \frac{4}{3} N_\mu^{2/3} r_d^* \right) \tag{A-5}$$

for $N_\mu \geqq 36\sqrt{2}(1 + 0.1\, N_{Re\infty}^{0.75})/N_{Re\infty}^{2}$. Therefore, the flow regime transition between the viscous flow and distorted particle flow can be given in terms of the viscosity number as shown in Figures 1 and 2. However, since in this regime the terminal velocity can be uniquely related to properties, Equation (A-5) can be rewritten in terms of the terminal velocity or the Reynolds number as

$$C_{D_\infty} = \frac{2\sqrt{2}}{3} N_\mu r_d^* v^*_{r_\infty} \left(\text{or } C_{D_\infty} = \frac{\sqrt{2}}{3} N_\mu N_{Re_\infty} \right) \quad \text{(A-6)}$$

The second expression is a special form based on the solution of the force balance between drag and gravity forces, and it may not be used in more general forced flow situations. As the size of bubbles further increases, the bubbles become spherical-cap shaped, and the drag coefficient reaches a constant value of $C_{D_\infty} = 8/3$. The transition from the distorted bubble regime to the spherical-cal bubble regime occurs at around $r_d^* = 2/N_\mu^{2/3}$. For a liquid drop, the drag coefficient can increase further according to Equation (A-5). However, eventually a droplet becomes unstable and disintegrates into smaller drops. This limit can be given by the well-known Weber number criterion and it corresponds to $r_d^* = 3/N_\mu^{2/3}$ and $C_D = 4$. The above results on the drag coefficient for single particles of solid, liquid, and gas are summarized in Figure 1.

By knowing the drag-law, $C_{D_\infty} = C_{D_\infty}(N_{Re_\infty})$, the terminal velocity can be calculated from Equation (A-2). In the viscous regime, the terminal velocity can be approximated by

$$v^*_{r_\infty} \simeq \frac{4.86}{r_d^*} [(1 + 0.08\, r_d^{*3})^{4/7} - 1] \quad \text{(A-7)}$$

On the other hand in the Newton's regime for solid particles, the drag coefficient is constant, therefore

$$v^*_{r_\infty} = 2.43\, r_d^{*1/2} \quad \text{(A-8)}$$

which holds for $r_d^* \geqq 34.54$.

For the distorted fluid particle regime, the terminal velocity reduces to a constant value of

$$v^*_{r_\infty} = \sqrt{2}/N_\mu^{1/3} \quad \text{(A-9)}$$

Hence, in this regime the relative velocity is independent of the fluid particle size. Further, for the spherical-cap bubble regime, the terminal velocity becomes

$$v^*_{r_\infty} = r_d^{*1/2} \quad \text{(A-10)}$$

These results are summarized in Figure 2.

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Electroforced Sedimentation of Thick Clay Suspensions in Consolidation Region

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It is shown that the slow settling rates of thickened Gairome clay slurries, when initial concentrations are sufficiently concentrated for settling due to the so-called consolidation mechanism from the beginning of settling, can be greatly enhanced by application of D.C. voltages. Equations for settling due to the consolidation mechanism are developed in view of the influence of the gravitational and the applied electric field. It is demonstrated that the settling rates increase remarkably with increasing electric field intensity. The settling rates and the porosity distributions in settling sediments calculated by the new equations compare favorably with experimental observations.

SCOPE

It has long been known that a charged region, called the electric double layer, appears at the interface between a suspended particle and the surrounding water. In an external D.C. electric field, the presence of the electric double layer causes effects on particle and liquid motion that are generally known as electrokinetic phenomena. Only a few industrial applications of these phenomena have been found, mainly in water clarification, cake and soil dehydration, and refining of natural clays. The works by Moulik et al. (1967) and Yukawa et al. (1971, 1972) imply a possibility of applications in filtering hard to filter materials such as bentonite clay and colloidal suspensions.

The present study is concerned with sedimentation due to consolidation of highly concentrated suspensions in D.C. electric fields. Its main objectives are to make clear the effects of electric fields on settling rates experimentally and to develop a basic mathematical method for analyzing electroforced settling behavior.

CONCLUSIONS AND SIGNIFICANCE

Sedimentation processes of highly concentrated slurries have been widely employed in sludge treatment and in the chemical and metallurgical industries. There exists a limiting slurry concentration for settling under the so-called consolidation mechanism. Settling rates due to consolidation are usually very low, and techniques for enhancing these rates could be useful.

The aggregate-floc model by Michaels and Bolger (1962) enables one to estimate the value of the limiting slurry concentration mentioned above. If the initial concentration of suspension is higher than the internal solid concentration of aggregates, only flocs, resulting from the perfect collapse of all aggregates, can exist in the suspension. Consequently, consolidation will occur from the beginning of sedimentation. In accordance with Michaels and Bolger's concept, preliminary settling experiments were carried out for Mitsukuri Gairome clay deionized water suspensions, and with the aid of a modified form of Richardson and Zaki's equation (Richardson and Zaki, 1954; Shirato et al., 1970), the limiting solid concentration was determined as